

From quantum foundations via natural language meaning to a theory of everything

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Abstract

In this paper we argue for a paradigmatic shift from ‘reductionism’ to ‘togetherness’. In particular, we show how interaction between systems in quantum theory naturally carries over to modelling how word meanings interact in natural language. Since meaning in natural language, depending on the subject domain, encompasses discussions within any scientific discipline, we obtain a template for theories such as social interaction, animal behaviour, and many others.

1 ...in the beginning was $\otimes$

No physicists! ...the symbol $\otimes$ above does not stand for the operation that turns two Hilbert spaces into the smallest Hilbert space in which the two given ones bilinearly embed. No category-theoreticians! ...neither does it stand for the composition operation that turns any pair of objects (and morphisms) in a monoidal category into another object, and that is subject to a horrendous bunch of conditions that guaranty coherence with the remainder of the structure. Instead, this is what it means:

$$\otimes \equiv \text{“togetherness”}$$

More specifically, it represents the togetherness of foo_1 and foo_2 without giving any specification of who/what foo_1 and foo_2 actually are. Differently put, it’s the new stuff that emerges when foo_1 and foo_2 get together. If they don’t like each other at all, this may be a fight. If they do like each other a lot, this may be a marriage, and a bit later, babies. Note that togetherness is vital for the emergence to actually take place, given that it is quite hard to either have a fight, a wedding, or a baby, if there is nobody else around.

It is of course true that in von Neumann’s formalisation of quantum theory the *tensor product of Hilbert spaces* (also denoted by $\otimes$) plays this role [39], giving rise to the emergent phenomenon of *entanglement* [20, 36]. And more generally, in category theory one can axiomatise *composition of*

objects (again denoted by $\otimes$) within a *symmetric monoidal category* [5], giving rise to elements that don't simply arise by pairing, just like in the case of the Hilbert space tensor product.

However, in the case of von Neumann's formalisation of quantum theory we are talking about a formalisation which, despite being widely used, its creator von Neumann himself didn't even like [33]. Moreover, in this formalism $\otimes$ only arises as a secondary construct, requiring a detailed description of foo_1 and foo_2 , whose togetherness it describes. What we are after is a 'foo-less' conception of $\otimes$. The composition operation $\otimes$ in symmetric monoidal categories heads in that direction. However, by making an unnecessary commitment to set-theory, it makes things unnecessarily complicated [16]. Moreover, while this operation is general enough to accommodate togetherness, it doesn't really tell us anything about it.

The title of this section is a metaphor aimed at confronting the complete disregard that the concept of togetherness has suffered in the sciences, and especially, in physics, where all of the effort has been on describing the individual, typically by breaking its description down to that of even smaller individuals. While, without any doubt, this has been a useful endeavour, it unfortunately has evolved in a rigid doctrine, leaving no space for anything else. The most extreme manifestation of this dogma is the use of the term 'theory of everything' in particle physics. We will provide an alternative conceptual template for a theory of everything, supported not only by scientific examples, but also by everyday ones.

Biology evolved from chopping up individual animals in laboratories, to considering them in the context of other other animals and varying environments. the result is the theory of evolution of species. Similarly, our current (still very poor) understanding of the human brain makes it clear that the human brain should not be studied as something in isolation, but as something that fundamentally requires interaction with other brains [30]. In contemporary audio equipment, music consists of nothing but a strings of zeros and ones. Instead, the entities that truly make up music are pitch, sound, rhythm, chord progression, crescendo, and so on. And in particular, music is not just a bag of these, since their intricate interaction is even more important than these constituents themselves. The same is true for film, where it isn't even that clear what it is made up from, but it does include such things as (easily replaceable) actors, decors, cameras, which all are part of a soup stirred by a director. But again, in contemporary video equipment, it is nothing but a string of zeros and ones.

In fact, everything that goes on in pretty much all modern devices is nothing but zeros and ones. While it was Turing's brilliance to realise that this could in fact be done, and provided a foundation for the theory of computability [38], this is in fact the only place where the zeros and ones are truly meaningful, in the form of a Turing machine. Elsewhere, it is nothing but a (universal) representation, with no conceptual qualities regarding the

subject matter.

2 Formalising togetherness 1: not there yet

So, how does one go about formalising the concept of togetherness? While we don't want an explicit description of the foo involved, we do need some kind of means for identifying foo. Therefore, we simply give each foo a name, say $A, B, C, \dots$. Then, $A \otimes B$ represents the togetherness of A and B . We also don't want an explicit description of $A \otimes B$, so how can we say anything about $\otimes$ without explicitly describing A , B and $A \otimes B$?

Well, rather than describing these systems themselves, we could describe their relationships. For example, in a certain theory togetherness could obey the following equation:

$$A \otimes A = A$$

That is, togetherness of two copies of something is exactly the same as a single copy, or in simpler terms, one is as good as two. For example, if one is in need of a plumber to fix a pipe, one only needs one. The only thing a second plumber would contribute is a bill for the time he wasted coming to your house. Obviously, this is not the kind of togetherness that we are really interested in, given that this kind adds nothing at all.

A tiny bit more interesting is the case that two is as good as three:

$$A \otimes A \otimes A = A \otimes A$$

e.g. when something needs to be carried on a staircase, but there really is only space for two people to be involved. Or, when A is female and $\bar{A}$ is male, and the goal is reproduction, we have:

$$A \otimes \bar{A} \otimes \bar{A} = A \otimes \bar{A}$$

(ignoring testosterone induced scuffles and the benefits of natural selection.)

We really won't get very far this manner. One way in which things can be improved is by replacing equations by inequalities. For example, while:

$$A = B$$

simply means that one of the two is redundant, instead:

$$A \leq B$$

can mean that from A we can produce B , and:

$$A \otimes B \leq C$$

can mean that from A and B together we can produce C , and:

$$A \otimes C \leq B \otimes C$$

can mean that in the presence of C from A we can produce B , i.e. that C is a catalyst.

What we have now is a so-called *resource theory*, that is, a theory which captures how stuff we care about can be interconverted [13]. Resource theories allow for quantitative analysis, for example, in terms of a *conversion rate*:

$$r(A \rightarrow B) := \sup \left\{ \frac{m}{n} \mid \underbrace{A \otimes \dots \otimes A}_n \leq \underbrace{B \otimes \dots \otimes B}_m \right\}$$

So evidently we have some genuine substance now.¹

3 Formalising togetherness 2: that's better

But we can still do a lot better. What a resource theory fails to capture (on purpose in fact) is the actual process that converts one resource into another one. So let's fix that problem, and explicitly account for processes.

In terms of togetherness, this means that we bring the fun foo_1 and foo_2 can have together explicitly in the picture. Let:

$$f : A \rightarrow B$$

denote some process that transforms A into B . Then, given two such processes f_1 and f_2 we can also consider their togetherness:

$$f_1 \otimes f_2 : A_1 \otimes A_2 \rightarrow B_1 \otimes B_2$$

Moreover, some processes can be sequentially chained:

$$g \circ f : A \rightarrow B \rightarrow C$$

We say 'some', since f has to produce B , in order for:

$$g : B \rightarrow C$$

to take place.

Now, here one may end up in a bit of a mess if one isn't clever. In particular, with a bit of thinking one quickly realises that one wants some equations to be obeyed, for example:

$$(f_1 \otimes f_2) \otimes f_3 = f_1 \otimes (f_2 \otimes f_3) \tag{1}$$

$$h \circ (g \circ f) = (h \circ g) \circ f \tag{2}$$

¹In fact, resource theories are currently a very active area of research in the quantum information and quantum foundations communities, e.g. the resource theories of entanglement [23], symmetry [21], and athermality [7].

and a bit more sophisticated, also:

$$(g_1 \otimes g_2) \circ (f_1 \otimes f_2) = (g_1 \circ f_1) \otimes (g_2 \circ f_2) \quad (3)$$

There may even be some more equations that one wants to have, but which ones? This turns out to be a very difficult problem. Too difficult in the light of our limited existence in this world. The origin of this problem is that we treat $\otimes$, and also $\circ$, as algebraic connectives, and that algebra has its roots in set-theory. The larger-than-life problem can be avoided in a manner that is equally elegant as it is simple.

To state that things are together, we just write them down together:

$$A \quad B$$

There really is no reason to adjoin the symbol $\otimes$ between them. Now, this A and B will play the role of an input or an output of processes transforming them. Therefore, it will be useful to represent them by a wire:

$$\left| \begin{array}{c} A \end{array} \right| \quad \left| \begin{array}{c} B \end{array} \right|$$

Then, a process transforming A into B can be represented by a box:

$$\begin{array}{c} |B \\ \boxed{f} \\ |A \end{array}$$

Togetherness of processes now becomes:

$$\begin{array}{cc} | & | \\ \boxed{f_1} & \boxed{f_2} \\ | & | \end{array}$$

and chaining processes becomes:

$$\begin{array}{c} | \\ \boxed{g} \\ | \\ \boxed{f} \\ | \end{array}$$

In particular, equations (1), (2) and (3) become:

$$\begin{array}{ccc} \begin{array}{c} | \\ \boxed{f_1} \\ | \end{array} & \begin{array}{c} | \\ \boxed{f_2} \\ | \end{array} & \begin{array}{c} | \\ \boxed{f_3} \\ | \end{array} = \begin{array}{c} | \\ \boxed{f_1} \\ | \end{array} \begin{array}{c} | \\ \boxed{f_2} \\ | \end{array} \begin{array}{c} | \\ \boxed{f_3} \\ | \end{array} \end{array} \quad \begin{array}{ccc} \begin{array}{c} | \\ \boxed{h} \\ | \\ \boxed{g} \\ | \\ \boxed{f} \\ | \end{array} & = & \begin{array}{c} | \\ \boxed{h} \\ | \\ \boxed{g} \\ | \\ \boxed{f} \\ | \end{array} \end{array}$$

$$\begin{array}{ccc} \begin{array}{c} | \\ \boxed{g_1} \\ | \\ \boxed{f_1} \\ | \end{array} & \begin{array}{c} | \\ \boxed{g_2} \\ | \\ \boxed{f_2} \\ | \end{array} & = \begin{array}{c} | \\ \boxed{g_1} \\ | \\ \boxed{f_1} \\ | \end{array} \begin{array}{c} | \\ \boxed{g_2} \\ | \\ \boxed{f_2} \\ | \end{array} \end{array}$$

That is, all equations have become tautologies!²

4 Anti-cartesian togetherness

One important kind of processes are *states*:



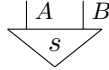
These are depicted without any inputs, where ‘no wire’ can be read as ‘nothing’ (or ‘no-foo’).³ The opposite notion is that of an *effect*, that is, a process without an output:



borrowing terminology from quantum theory.⁴

We can now identify those theories in which togetherness doesn't yield anything new. Life in such a world is pretty lonely...

Definition 4.1. A theory of togetherness is *cartesian* if each state:



decomposes as follows:

$$\begin{array}{c} |A|B \\ \hline \triangle s \end{array} = \begin{array}{c} |A| \\ \hline \triangle s_1 \end{array} \begin{array}{c} |B| \\ \hline \triangle s_2 \end{array}$$

So cartesianness means that all possible realisations of two foo-s can be achieved by pairing realisations of the individual foo-s involved. In short, a whole can be described in term of its parts, rendering togetherness a void concept. So very lonely and indeed... But, wait a minute. Why is it then the case that so much of traditional mathematics follows this cartesian template, and that even category theory for a long time has followed a strict cartesian stance? Beats me. Seriously...beats me!

Anyway, an obvious consequence of this is that for those areas where togetherness is a genuinely non-trivial concept, traditional mathematical structures aren't always that useful. That is maybe why social sciences don't make much use of any kind of modern pure mathematics.

And now for something completely different:

²A more extensive discussion of this bit of magic can be found in [16, 12, 15].

³That we use triangles for these rather than boxes is inspired by the Dirac notation which is used in quantum theory. Please consult [10, 16, 15] for a discussion.

⁴Examples of these include ‘tests’ [15].

Definition 4.2. A theory of togetherness is *anti-cartesian* if for each A there exists A^* , a special state $\cup$ and a special effect $\cap$:

$$\begin{array}{c} |A^*|A \\ \hline \cup \end{array} \quad \text{and} \quad \begin{array}{c} \cap \\ \hline |A|A^* \end{array}$$

which are such that the following equation holds:

$$\begin{array}{c} \cap \\ \hline |A| \begin{array}{c} |A^*|A \\ \hline \cup \end{array} \end{array} = |A \quad (4)$$

The reason for ‘anti’ in the name of this kind of togetherness is the fact that when a theory of togetherness is both cartesian and anti-cartesian, then it is nothing but a theory of absolute death, i.e. it describes a world in which nothing ever happens. Indeed, we have:

$$\begin{array}{c} \text{anti-cartesian} \\ \swarrow \\ | \end{array} = \begin{array}{c} \cap \\ \hline | \begin{array}{c} |A^*|A \\ \hline \cup \end{array} \end{array} \begin{array}{c} \text{cartesian} \\ \swarrow \\ | \end{array} = \begin{array}{c} \cap \\ \hline | \begin{array}{c} \cup_1 \\ \hline \cap \end{array} \end{array} \begin{array}{c} \cup_2 \\ \hline | \end{array} = \begin{array}{c} \cup_2 \\ \hline \begin{array}{c} \cap \\ \hline \begin{array}{c} \cup_1 \\ \hline \cap \end{array} \end{array} \end{array} = \begin{array}{c} \cup_2 \\ \hline | \end{array} \begin{array}{c} t \\ \hline | \end{array}$$

That is, the *identity* is a constant process, always outputting the state $\cup_2$, independent of what the input is. And if that isn’t weird enough, any arbitrary process f does the same:

$$\begin{array}{c} | \\ \hline \boxed{f} \end{array} = \begin{array}{c} | \\ \hline \boxed{f} \end{array} = \begin{array}{c} \cup_2 \\ \hline \begin{array}{c} t \\ \hline \boxed{f} \end{array} \end{array} = \begin{array}{c} \cup_2 \\ \hline \begin{array}{c} t' \\ \hline | \end{array} \end{array}$$

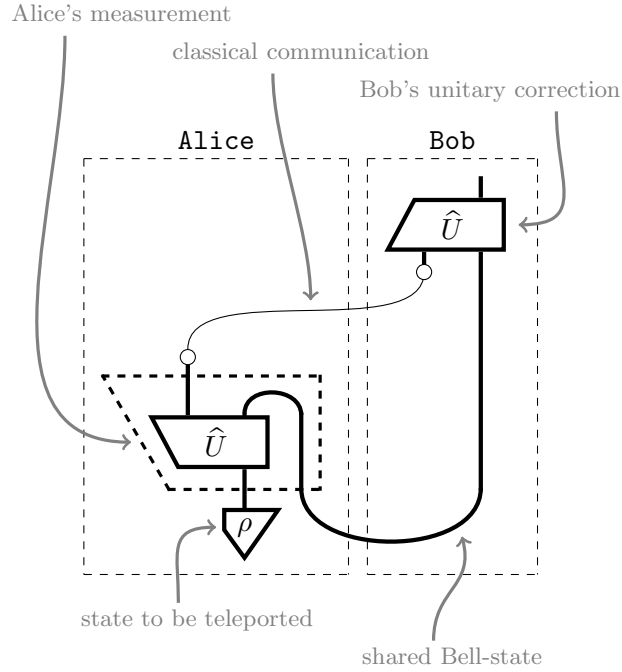
Therefore, any anti-cartesian theory of togetherness that involves some aspect of change cannot be cartesian, and hence will have interesting stuff emerging from togetherness.⁵

5 Example 1: quantum theory

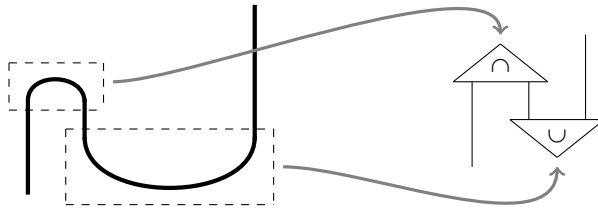
Anti-cartesian togetherness is a very particular alternative to cartesian togetherness (contra any theory that fails to be cartesian). So one may wonder

⁵Many more properties of anti-cartesian togetherness can be found in [15].

whether there are any interesting examples. And yes, there are! One example is *quantum entanglement* in quantum theory. That is in fact where the author's interest in anti-cartesian togetherness started [9, 1, 10].⁶ As shown in these papers, equation (4) pretty much embodies the phenomenon of quantum teleportation [6]. The full-blown description of quantum teleportation goes as follows [17, 18, 15]:

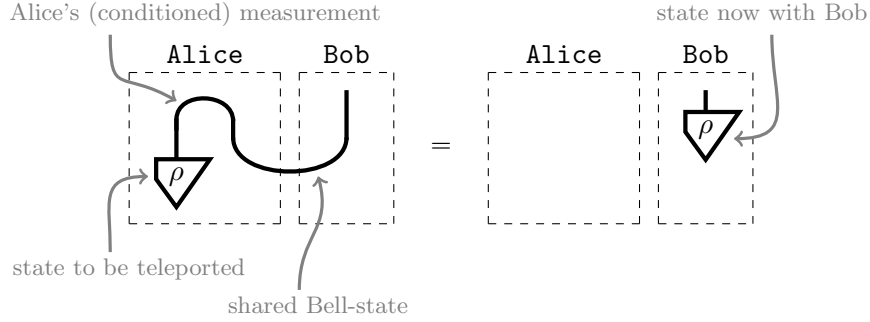


It is not important to fully understand the details here. What is important is to note that the bit of this diagram corresponding to equation (4) is the bold wire which zig-zags through it:

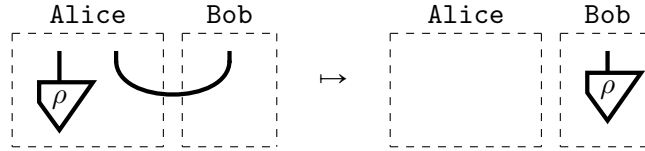


The thin wires and the boxes labelled $\hat{U}$ are related to the fact that quantum theory is non-deterministic. By conditioning on particular measurement outcomes, teleportation simplifies to [15]:

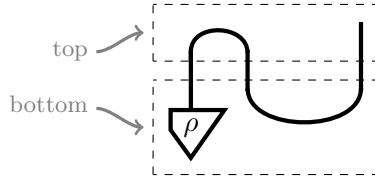
⁶Independently, similar insights appeared in [3, 26].



Equality of the left-hand-side and of the right-hand-side follows directly from equation (4). While in this picture we eliminated quantum non-determinism by conditioning on a measurement outcome, there still is something very ‘quantum’ going on here: Alice’s (conditioned) measurement is nothing like a passive observation, but a highly non-trivial intervention that makes Alice’s state ρ appear at Bob’s side:



Let’s analyse more carefully what’s going on here by explicitly distinguishing the top layer and the bottom layer of this diagram:



The bottom part:

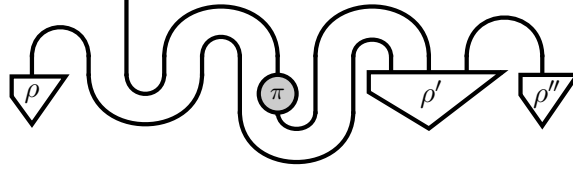


consists of the state ρ together with a special $\cup$ -state, while the top part:

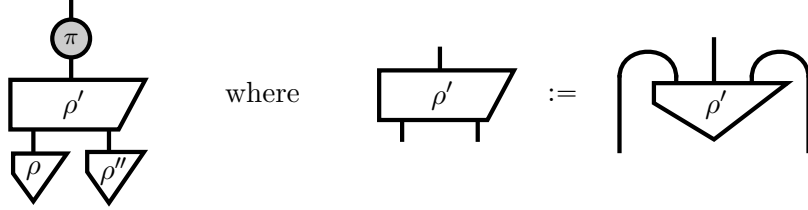


includes the corresponding $\cap$ -effect, as well as an output. By making the bottom part and the top part interact, and, in particular, the $\cup$ and the $\cap$, the state ρ ends up at the output of the top part.

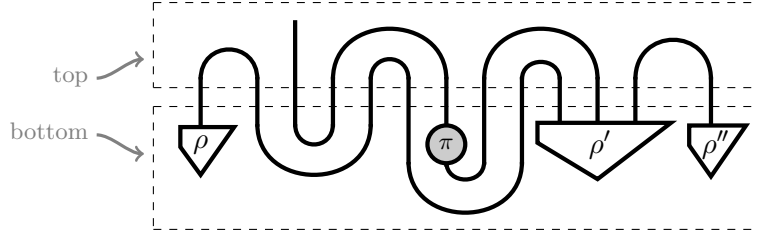
A more sophisticated variation on the same theme makes it much clearer which mechanism is going on here. Using equation (4), the diagram:



reduces to:



The grey dot labeled π is some (at this point not important) unitary quantum operation [15]. Let us again consider the bottom and top parts:



The top part is a far more sophisticated measurement consisting mainly of $\cap$ -s. Also the bottom part is a lot more sophisticated, involving many $\cup$ -s. These now cause a highly non-trivial interaction of the three states ρ , ρ' and ρ'' . Why we have chosen this particular example will become clear in the next section. What is important to note is that the overall state and overall effect have to be chosen in a very particular way to create the desired interaction, similarly to an old-fashion telephone switchboard that has to be connected in a very precise manner in order to realise the right connection.

6 Example 2: natural language meaning

Another example of anti-cartesian togetherness is the manner in which word meanings interact in natural language! Given that logic originated in natural language, when Aristotle analysed arguments involving ‘and’, ‘if...then’, ‘or’, etc., anti-cartesianness can be conceived as some new kind of logic!⁷ So what are $\cup$ and $\cap$ in this context?

In order to understand what $\cap$ is, we need to understand the mathematics of grammar. The study of the mathematical structure of grammar has indicated that the fundamental things making up sentences are not the

⁷A more detailed discussion is in [11].

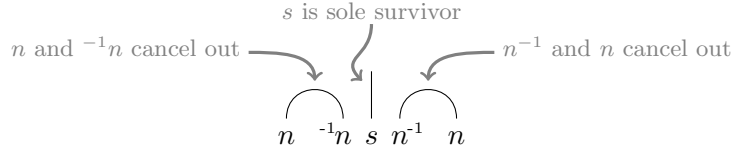
words, but some atomic grammatical types, such as the noun-type and the sentence-type [2, 4, 27]. The transitive verb-type is not an atomic grammatical type, but a composite made up of two noun-types and one sentence-type. Hence, particularly interesting here is that atomic doesn't really mean smallest...

On the other hand, just like in particle physics where we have particles and anti-particles, the atomic types include types as well as anti-types. But unlike in particle physics, there are two kinds of anti-types, namely left ones and right ones. This makes language even more non-commutative than quantum theory!

All of this becomes much clearer when considering an example. Let n denote the atomic *noun*-type and let ^{-1}n and n^{-1} be the corresponding anti-types. Let s denote the atomic *sentence*-type. Then the non-atomic *transitive verb*-type is $^{-1}n \cdot s \cdot n^{-1}$. Intuitively, it is easy to understand why. Consider a transitive verb, like 'hate'. Then, simply saying 'hate' doesn't convey any useful information, until, we also specify 'whom' hates 'whom'. That's exactly the role of the anti-types: they specify that in order to form a meaningful sentence, a noun is needed on the left, and a noun is needed on the right:

$$\underbrace{Alice}_n \quad \underbrace{hates}_{^{-1}n \cdot s \cdot n^{-1}} \quad \underbrace{Bob}_n$$

Then, n and ^{-1}n cancel out, and so do n^{-1} and n . What remains is s , confirming that 'Alice hates Bob' is a grammatically well-typed sentence. We can now depict the cancelations as follows:



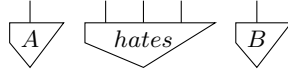
and bingo, we found $\cap$!

While the mathematics of sentence structure has been explored now for some 80 years, the fact that $\cap$ -s can account for grammatical structure is merely a 15 years old idea [28]. So what are the $\cup$ -s? That is an even more recent story in which we were involved, and in fact, for which we took inspiration from the story of the previous section [8]. While $\cap$ -s are about grammar, $\cup$ -s are about meaning.

The *distributional* paradigm for natural language meaning states that meaning can be represented by vectors in a vector space [37]. Until recently, grammatical structure was essentially ignored in doing so, and in particular, there was no theory for how to compute the meaning of a sentence, given the meanings of its words. Our new *compositional distributional model of*

meaning of [19] does exactly that.⁸

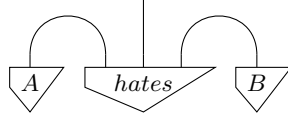
In order to explain how this compositional distributional model of meaning works, let's get back to our example. Since we have grammatical types around, the meaning vectors should respect grammatical structure, that is, the vectors representing compound types should themselves live in compound vector spaces. So the string of vectors representing the word meanings of our example would look as follows:



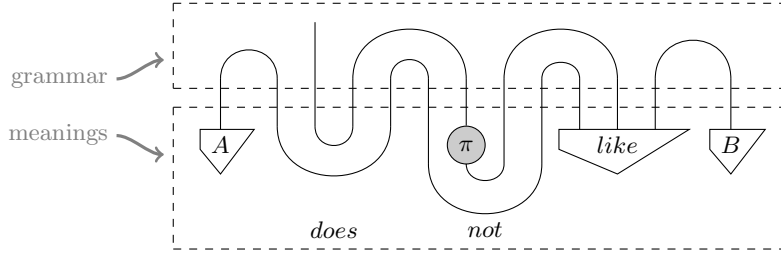
Now we want to put forward a new hypothesis:

Grammar is all about how word meanings interact.

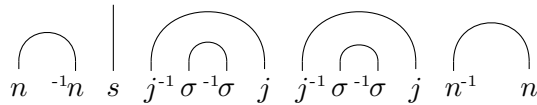
Inspired by the previous section, this can be realised as follows:



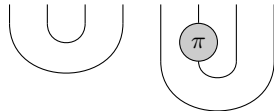
where the $\cap$ -s are now interpreted in exactly the same manner as in the previous section. And here is a more sophisticated example:



where the π -labeled grey circle should now be conceived as negating meaning [19]. The grammatical structure is here:

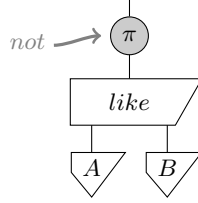


It is simply taken from a textbook such as [29], the meanings of *Alice*, *likes* and *Bob* can be automatically generated from some corpus, while the meanings of *does* and *not* are just cleverly chosen to be [32, 19]:



⁸...and has meanwhile outperformed other attempts in several benchmark natural language processing (NLP) tasks [22, 25].

In the previous section we already saw that in this way we obtain:



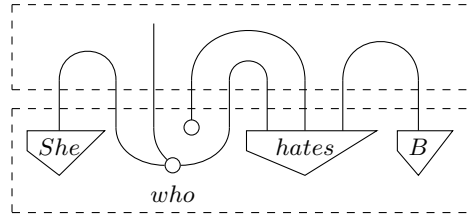
This indeed captures the intended meaning:

$$\text{not}(\text{like}(\text{Alice}, \text{Bob}))$$

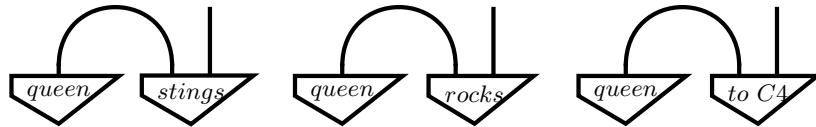
where we can think of *like* as being a predicate and *not* as being itself.

So an interesting new aspect of the last example is that some of the meaning vectors of words are simply cleverly chosen, and in particular, involve $\cup$ -s. Hence, we genuinely exploit full-blown anti-cartesianess. What anti-cartesianess does here is making sure that the transitive verb *likes* ‘receives’ *Alice* as its object. Note also how *not* does pretty much the same as *does*, guiding word meanings through the sentence, with, of course, one very important additional task: negating the sentence meaning.

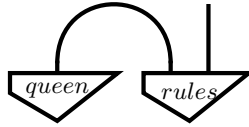
The cautious reader must of course have noticed that in the previous section we used thick wires, while here we used thin ones. Also, the dots in the full-blown description of quantum teleportation, which represent classical data operations, have vanished in this section. Meanwhile, thick wires as well as the dots all of these have acquired a vary natural role in a more refined model of natural language meaning. The dots allow to cleverly choose the meanings of relative pronouns [34, 35]:



Thick wires (representing density matrices, rather than vectors [15]) allow to encode word ambiguity as *mixedness* [31, 24]. For example, the different meanings of the word *queen* (a rock band, a person, a bee, a chess piece, or a drag —). Mixedness vanishes when providing a sufficient string of words that disambiguates that meaning, e.g.:



while in the case of:



we need more disambiguating words, since *queen* can still refer to a person, a rock band, as well as a drag queen.

7 Meaning is everything

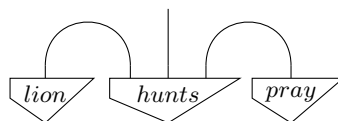
The distributional model of meaning [37] is very useful in that it allows for automation, given a substantially large corpus of text. However, from a conceptual point of view it is far from ideal. So one may ask the question:

What is meaning?

One may try to play around with a variety of mathematical structures. The method introduced in [19] doesn't really depend on how one models meaning, as long as we stick to anti-cartesian togetherness, or something sufficiently closely related [14]. It is an entertaining exercise to play around with the idea of what possibly could be the ultimate mathematical structure that captures meaning in natural language, until one realises that meaning in natural language truly encompasses *everything*. Indeed, we use language to talk about everything, e.g. logic, life, biology, physics, social behaviours, politics, so the ultimate model of meaning should encompass all of these fields. So, a theory of meaning in natural language is actually a theory of everything! Can we make sense of the template introduced in the previous section for meaning in natural language, as one for ... everything?

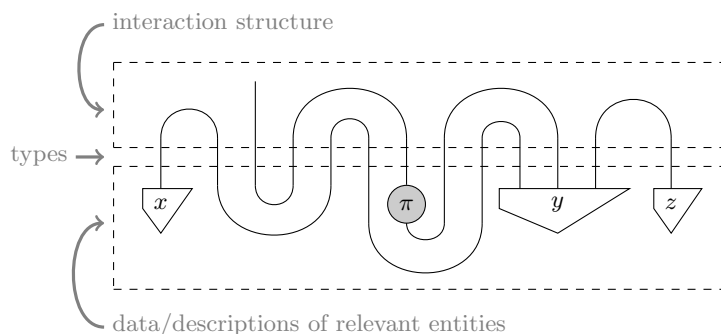
Let us first investigate, whether the general distributional paradigm can be specialised to the variety of subject domains mentioned above. The manner in which the distributional model works, is that meanings are assigned relative to a fixed chosen set of context words. The meaning vector of any word then arises by counting the number of occurrences of that word in the close neighbourhood of each of the context words, within a large corpus of text. One can think of the context words as attributes, and the relative frequencies as the relevance of an attribute for the word. Simply by specialising the context words and the corpus, one can specialise to a certain subject domain. For example, if one is interested in social behaviours then the corpus could consist of social networking sites, and the context words could be chosen accordingly. This pragmatic approach allows for quantitative analysis, just like the compositional distributional model of [19].

Here's another example:



Here the meaning of pray could include specification of the available pray, and then the meaning of the sentence would capture the survival success of the lion, given the nature of the available pray. All together, the resulting meaning is the result of the interaction between a particular hunter, a particular pray, and the intricacies of the hunting process, which may depend on the particular environment in which it is taking place. It should be clear that again this situation is radically non-cartesian.

Of course, if we now consider the example of quantum theory from two sections ago, the analogues to grammatical types are system types i.e. a specification of the kinds (incl. quantity) of systems that are involved. So it makes sense to refine the grammatical types according to the subject domain. Just like nouns in physics would involve specification of the kinds of systems involved, in biology, for example, this could involve specification of species, population size, environment, availability of food etc. Correspondingly, the top part would not just be restricted to grammatical interaction, but also domain specific interaction, just like in the case of quantum theory. All together, what we obtain is the following picture:



as a (very rough) template for a theory of everything.

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References

- [1] S. Abramsky and B. Coecke. A categorical semantics of quantum protocols. In *Proceedings of the 19th Annual IEEE Symposium on Logic in Computer Science (LICS)*, pages 415–425, 2004. arXiv:quant-ph/0402130.

- [2] K. Ajdukiewicz. Die syntaktische konnexität. *Studia Philosophica*, 1:1–27, 1937.
- [3] J. C. Baez. Quantum quandaries: a category-theoretic perspective. In D. Rickles, S. French, and J.T. Saatsi, editors, *The Structural Foundations of Quantum Gravity*, pages 240–266. Oxford University Press, 2006. arXiv:quant-ph/0404040.
- [4] Y. Bar-Hillel. A quasiarithmetical notation for syntactic description. *Language*, 29:47–58, 1953.
- [5] J. Benabou. Categories avec multiplication. *Comptes Rendus des Séances de l’Académie des Sciences. Paris*, 256:1887–1890, 1963.
- [6] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters. Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels. *Physical Review Letters*, 70(13):1895–1899, 1993.
- [7] F. G. S. L. Brandão, M. Horodecki, J. Oppenheim, J. M. Renes, and R. W. Spekkens. The resource theory of quantum states out of thermal equilibrium. *Physical Review Letters*, 111:250404, 2013.
- [8] S. Clark, B. Coecke, E. Grefenstette, S. Pulman, and M. Sadrzadeh. A quantum teleportation inspired algorithm produces sentence meaning from word meaning and grammatical structure. arXiv:1305.0556, 2013.
- [9] B. Coecke. The logic of entanglement. An invitation. Technical Report RR-03-12, Department of Computer Science, Oxford University, 2003.
- [10] B. Coecke. Kindergarten quantum mechanics. In A. Khrennikov, editor, *Quantum Theory: Reconsiderations of the Foundations III*, pages 81–98. AIP Press, 2005. arXiv:quant-ph/0510032.
- [11] B. Coecke. The logic of quantum mechanics – take II. arXiv:1204.3458, 2012.
- [12] B. Coecke. An alternative Gospel of structure: order, composition, processes. In C. Heunen, M. Sadrzadeh, and E. Grefenstette, editors, *Quantum Physics and Linguistics. A Compositional, Diagrammatic Discourse*, pages 1 – 22. Oxford University Press, 2013. arXiv:1307.4038.
- [13] B. Coecke, T. Fritz, and R. W. Spekkens. A mathematical theory of resources. *Information and Computation*, to appear, 2014. arXiv:1409.5531.
- [14] B. Coecke, E. Grefenstette, and M. Sadrzadeh. Lambek vs. Lambek: Functorial vector space semantics and string diagrams for Lambek calculus. *Annals of Pure and Applied Logic*, 164:1079–1100, 2013.

- [15] B. Coecke and A. Kissinger. *Picturing Quantum Processes. A First Course in Quantum Theory and Diagrammatic Reasoning*. Cambridge University Press, 2016.
- [16] B. Coecke and É. O. Paquette. Categories for the practicing physicist. In B. Coecke, editor, *New Structures for Physics*, Lecture Notes in Physics, pages 167–271. Springer, 2011. arXiv:0905.3010.
- [17] B. Coecke, É. O. Paquette, and D. Pavlović. Classical and quantum structuralism. In S. Gay and I. Mackie, editors, *Semantic Techniques in Quantum Computation*, pages 29–69. Cambridge University Press, 2010. arXiv:0904.1997.
- [18] B. Coecke and S. Perdrix. Environment and classical channels in categorical quantum mechanics. In *Proceedings of the 19th EACSL Annual Conference on Computer Science Logic (CSL)*, volume 6247 of *Lecture Notes in Computer Science*, pages 230–244, 2010. Extended version: arXiv:1004.1598.
- [19] B. Coecke, M. Sadrzadeh, and S. Clark. Mathematical foundations for a compositional distributional model of meaning. In J. van Benthem, M. Moortgat, and W. Buszkowski, editors, *A Festschrift for Jim Lambek*, volume 36 of *Linguistic Analysis*, pages 345–384. 2010. arxiv:1003.4394.
- [20] A. Einstein, B. Podolsky, and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Physical review*, 47(10):777, 1935.
- [21] G. Gour and R. W. Spekkens. The resource theory of quantum reference frames: manipulations and monotones. *New Journal of Physics*, 10:033023, 2008.
- [22] E. Grefenstette and M. Sadrzadeh. Experimental support for a categorical compositional distributional model of meaning. In *The 2014 Conference on Empirical Methods on Natural Language Processing.*, pages 1394–1404, 2011. arXiv:1106.4058.
- [23] R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki. Quantum entanglement. *Reviews of Modern Physics*, 81:865–942, 2009. arXiv:quant-ph/0702225.
- [24] D. Kartsaklis. *Compositional Distributional Semantics with Compact Closed Categories and Frobenius Algebras*. PhD thesis, University of Oxford, 2014.

- [25] D. Kartsaklis and M. Sadrzadeh. Prior disambiguation of word tensors for constructing sentence vectors. In *The 2013 Conference on Empirical Methods on Natural Language Processing.*, pages 1590–1601. ACL, 2013.
- [26] L. H. Kauffman. Teleportation topology. *Optics and Spectroscopy*, 99:227–232, 2005.
- [27] J. Lambek. The mathematics of sentence structure. *American Mathematics Monthly*, 65, 1958.
- [28] J. Lambek. Type grammar revisited. *Logical Aspects of Computational Linguistics*, 1582, 1999.
- [29] J. Lambek. From word to sentence. *Polimetrika, Milan*, 2008.
- [30] M. D. Lieberman. *Social: Why our Brains are Wired to Connect*. Oxford University Press, 2013.
- [31] R. Piedeleu. Ambiguity in categorical models of meaning. Master’s thesis, University of Oxford, 2014.
- [32] A. Preller and M. Sadrzadeh. Bell states and negative sentences in the distributed model of meaning. *Electronic Notes in Theoretical Computer Science*, 270(2):141–153, 2011.
- [33] M. Redei. Why John von Neumann did not like the Hilbert space formalism of quantum mechanics (and what he liked instead). *Studies in History and Philosophy of Modern Physics*, 27(4):493–510, 1996.
- [34] M. Sadrzadeh, S. Clark, and B. Coecke. The Frobenius anatomy of word meanings I: subject and object relative pronouns. *Journal of Logic and Computation*, 23:1293–1317, 2013. arXiv:1404.5278.
- [35] M. Sadrzadeh, S. Clark, and B. Coecke. The Frobenius anatomy of word meanings II: possessive relative pronouns. *Journal of Logic and Computation*, page exu027, 2014.
- [36] E. Schrödinger. Discussion of probability relations between separated systems. *Cambridge Philosophical Society*, 31:555–563, 1935.
- [37] H. Schütze. Automatic word sense discrimination. *Computational linguistics*, 24(1):97–123, 1998.
- [38] A. M. Turing. On computable numbers, with an application to the Entscheidungsproblem. *Proceedings of the London Mathematical Society*, 42:230–265, 1937.

- [39] J. von Neumann. *Mathematische grundlagen der quantenmechanik*. Springer-Verlag, 1932. Translation, *Mathematical foundations of quantum mechanics*, Princeton University Press, 1955.